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1990 J. Phys. A: Math. Gen. 23 L57

(http://iopscience.iop.org/0305-4470/23/2/003)

J. Phys. A: Math. Gen. 23 (1990) L57-L60. Printed in the UK

LETTER TO THE EDITOR

Can GRW theory be tested by experiments on SQUIDS?

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Received 16 October 1989

Abstract. A recent theory shows that a collapse of the wavefunction representing a macroscopic pointer would result if one of its component particles were spontaneously localised. The consequences of such microscopic localisation for a SQUID operating in the macroscopic quantum regime is discussed.

Ghirardi *et al* [1] have proposed a modification of quantum mechanics (now known as GRW theory) in which all elementary particles are assumed to undergo random occasional localisations. Another version of this theory, in which the localisation proceeds continuously, has been recently developed by Pearle [2]. In both cases the probability of a particular microscopic particle being localised is so low that its time evolution is practically the same as that predicted by the time-dependent Schrödinger equation. However, it can be shown that a macroscopic body (such as the pointer of a measuring instrument) which is composed of a very large number of particles is likely to exist in a delocalised quantum state for only a very short time, after which it collapses into a localised state like that predicted by classical mechanics and always observed experimentally. This then provides a possible solution to the 'Schrödinger cat' problem in the quantum theory of measurement, although at considerable cost: the values of two parameters have to be postulated which would probably be new fundamental constants of nature, unless the speculation [2] that they are related to quantum gravity bears fruit. The general possibility of the breakdown of quantum mechanics in the macroscopic regime has been discussed by Leggett [3] who has suggested that this idea could be tested by performing experiments on superconducting quantum interference devices (SQUIDS). The purpose of the present letter is to investigate whether such SQUID experiments would constitute a test of the particular macroscopic breakdown predicted by GRW theory.

Following the exposition of GRW theory by Bell [4], the wavefunction $\psi(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N)$ of an N-particle system is changed by a spontaneous localisation of the ith particle into

$$\psi'(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) = [j(\boldsymbol{x} - \boldsymbol{r}_i)/R_i(\boldsymbol{x})]\psi(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) \qquad (1)$$

where $j(\boldsymbol{x})$ is localised in space around $\boldsymbol{x} = \boldsymbol{0}$ and is taken by GRW to be a Gaussian of width 10^{-7} m. $R_i(\boldsymbol{x})$ is chosen so that ψ' is normalised and the probability distribution function for the position of the collapse centre $\boldsymbol{x}$ is assumed to be $|R_i|^2$. It follows that localisation is probable only at points where the initial wavefunction ψ is significantly large.

We now consider a macroscopic body composed of particles whose relative positions are well determined, e.g. a solid insulator. In this case, even if the localisation probability for a single particle is very small (GRW assume one localisation every 10^{15} s), that for the localisation of one or other of the component particles in the body is quite large (once in every 10^{-5} s for a body composed of 10^{20} particles). Consider now the case where the initial state ψ is a linear combination of two spatially separated states, as would be the case if the body were the pointer of an instrument used to make a measurement of a microscopic system in an appropriate quantum state. The initial wavefunction is now

$$A\psi_{\mathrm{R}}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) + B\psi_{\mathrm{L}}(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_N) \tag{2}$$

where ψ_{R} and ψ_{L} correspond to the two spatially separated pointer states, and A and B are constants. After a GRW localisation of the ith particle, the wavefunction acquires a form analogous to that set out in equation (1). However, because the localisation probability is appreciable only at points where ψ is significantly large, the point $\boldsymbol{x}$ must lie either within the region occupied by ψ_{R}, in which case the product of the localisation function with ψ_{L} is negligibly small or vice versa. It follows that GRW localisation of one of its component particles causes a collapse of the wavefunction describing the whole pointer. Furthermore, it can be easily shown that the probabilities of the two possible outcomes are just $|A^2|$ and $|B^2|$, as predicted by the conventional quantum theory of measurement. A more detailed discussion of the application of GRW theory to the motion of a solid body such as a pointer is given by Diosi [5].

Before getting to SQUID, we first consider the case of a simple superconducting circuit. The BCS wavefunction of the superconducting state can be written as [6]

$$\psi = \psi_{\boldsymbol{k}_1}\psi_{\boldsymbol{k}_2} \ldots \psi_{\boldsymbol{k}_N} \exp[\mathrm{i}S(\boldsymbol{r})] \tag{3}$$

where $\psi_{\boldsymbol{k}_j}$ represents the wavefunction of a (possibly partially occupied) Cooper pair composed of electrons with wavevectors plus and minus $\boldsymbol{k}_j$ and $S(\boldsymbol{r})$ is the macroscopic phase associated with the supercurrent. An important point to note is that the $\psi_{\boldsymbol{k}_i}$ are each delocalised over the entire superconductor. The maximum consequence of a GRW localisation of one electron would therefore be the break up of one of the Cooper pairs, which would result in the supercurrent being reduced by about one part in 10^{20}. Even if this happens once every 10^{-5} s, the resulting decay in the supercurrent would be well below the experimental detection limit of around one part in 10^{13} per second [7]. Moreover, the above argument assumes that the supercurrent decays continuously as the localisations proceed, whereas a more realistic model would have to take into account the possibility of recreation of Cooper pairs which would lower the observability of the localisation process even further. We can therefore draw the non-trivial conclusion that GRW theory is not incomptible with the very existence of superconductivity.

Turning now to the SQUID, a typical device consists of a ring of superconductor interrupted by a Josephson junction or other 'weak link'. As a result, the potential energy of the system is a function of the magnetic flux enclosed in the ring and (provided the strength of the link and an external bias field are appropriately chosen) this has the form of a double-well potential. Moreover, it can be shown [3] that the quantum behaviour of the device is governed by a Schrödinger-type equation in flux space with the particle mass replaced by the electrical capacitance of the junction. Clearly then the energy eigenstates of the system are delocalised across the potential barrier and if the ground-state energy is significantly below the barrier height, the wavefunction of the state can be closely approximated by

$$\psi(1, 2, \ldots, N) = 2^{-1/2}(\psi_1 + \psi_2) \tag{4}$$

where ψ_1 and ψ_2 correspond to the flux being in the vicinity of one or other of the potential minima. The potential minima are separated in flux space by a distance of the order of one flux quantum and this corresponds to a difference in the circulating supercurrent of around 10^{-7} A, so in this sense the component states are macroscopically separated and, moreover, the form of the wavefunction (4) is similar to that of the delocalised pointer (2). SQUID states such as these would therefore seem to be highly relevant to the question of the applicability of quantum mechanics to the macroscopic regime. We now note that the only difference between ψ_1 and ψ_2 is that they correspond to different supercurrents; we can therefore use (3) to rewrite (4) as

$$\psi = \psi_{k1}\psi_{k2}\ldots\psi_{kN}\{\exp[iS_1(\boldsymbol{r})]+\exp[iS_2(\boldsymbol{r})]\}. \tag{5}$$

Following (1), the effect of GRW localisation on this, as for any other wavefunction, is to multiply it by the localisation function $j(\boldsymbol{x}-\boldsymbol{r}_i)/R(\boldsymbol{x})$. However, it is clear that, unlike its effect on the pointer state (2), neither of the macroscopically separated states in (5) is selected by this process and the only result is a break up of one of the Cooper pairs, as was the case for the simple superconducting state (3). The localisation may have some effect on the coherence of the linear superposition but this will presumably be similar to that arising from other dissipative processes such as those associated with the parallel normal resistance of the weak link. Leggett [8] has discussed the influence of dissipation on the possibility of observing the quantum coherence of states such as (5) in the presence of dissipation and has concluded that if the potential minima are well separated in flux space, dissipation will be unimportant provided the shunting resistance is much greater than about 7 kΩ (the characteristic quantum resistance). We saw in the previous paragraph that the decay time associated with GRW localisation is about 10^{15} s, which corresponds to a resistance of 10^{30} Ω, assuming a weak-link capacitance of 10^{-15} F (which is the order required for observable macroscopic quantum coherence [8]). It therefore follows that GRW localisation should have a negligible effect on the behaviour of the SQUID, even when operating in the macroscopically coherent quantum regime. It is interesting to note that this conclusion still holds in the case of a two-hole SQUID where the potential minima correspond to different currents circulating around the spatially separated holes. It would be tempting to expect in this case that a real-space localisation would result in a collapse into one or other of these two current states. However, this ignores the fact that the superconducting wavefunction is delocalised over the whole conductor whatever the local value of the current, so that exactly the same arguments apply as in the other cases discussed above.

We therefore conclude that GRW theory would not cause a reduction of the wavefunction representing a coherent superposition of SQUID states macroscopically separated in flux space and that the GRW postulates could be neither verified nor falsified by such experiments. However, several caveats have to be made. First, we have assumed that GRW theory refers literally to real space localisations. If, as has been suggested [4], the principle might be more correctly applied to field variables, then if one of these were to be the flux enclosed in a SQUID ring, GRW localisation could lead to a collapse in this case; however, the SQUID might well be equivalent to a single particle in this representation, so such a localisation could well be unobservably rare. Secondly, any proposed experimental test would have to take into account how the state of the SQUID was to be actually measured. Peres [9] describes an idealised set-up in which the flux variable is associated with the momentum of a pointer and the canonically-conjugate charge varable with its position. Clearly, GRW theory could cause a localisation of the pointer in such a set-up and this in turn would result in a collapse of the wavefunction describing the whole system. Finally, we should note that the experimental detection of quantum delocalisation in SQUID presents many formidable difficulties [10] and has not yet been achieved. As this is probably the easiest area in which to study the quantum mechanics of macroscopic objects, the prospects of experiments that would actually test GRW theory being performed in the near future must be very slight.

I am grateful to Philip Pearle for interesting discussions.

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